METHOD OF TRANSFORMATION

No general method can be prescribed for integrating a function. We have to take a (सहारा) recourse to different method according to different circumstances. Whatever methods of integration are available to us, they got only one objective, and this is to transform the given integral into any such function which resembles with that given in the standard form. In this topic we will simplify and transform it in form of sum or difference of two or more functions; can be put in standard form i.e, so that its integral can be easily found out.

Q) Evaluate, $\int \sin 2x \cdot \sin 3x \, dx$

Solution :- $\frac{1}{2} \int 2 \sin 2x \cdot \sin 3x$

$= \frac{1}{2} \int [\cos(2x-3x) - \cos(2x+3x)]$

$= \frac{1}{2} \int [\cos x - \cos 5x]$

$I = \frac{1}{2} [\cos x - \cos 5x]$

$= \frac{1}{2} \left[\sin x - \frac{\sin 5x}{5}\right]$ Ans

Q) Evaluate, $\int \sin x \cdot \sin 2x \cdot \sin 3x \, dx$

Solution, $\int (\sin x \cdot \sin 2x \cdot \sin 3x) dx$

$= \frac{1}{2} \int (2 \sin x \cdot \sin 2x \cdot \sin 3x)$

$= \frac{1}{2}(2\sin 2x \cdot \sin 2x) \cdot \sin 3x$

$= \frac{1}{2}[\cos x - \cos 3x] \sin 2x$

$= \frac{1}{2}[\cos x \cdot \sin 3x - \cos 3x \cdot \sin 3x]$

$= \frac{1}{2}\left[\frac{1}{2} 2\cos x \cdot \sin 3x - 2\frac{1}{2} \cos 3x \cdot \sin 3x\right]$

$= \frac{1}{2} \cdot \frac{1}{2}[\sin 4x + \sin 2x - \sin 6x + 0]$

$= \frac{1}{4}[\sin 4x + \sin 2x - \sin 6x]$

On integrating we get,

$= \frac{1}{4}[\sin 4x + \sin 2x - \sin 6x]$

$= \frac{1}{4}\left[-\frac{\cos 4x}{4} + \frac{\cos 2x}{2} + \frac{\cos 6x}{6}\right]$

$= \frac{1}{4}\left(\frac{\cos 6x}{6} - \frac{\cos 2x}{2} - \frac{\cos 4x}{4}\right)$

Ans

Q) Evaluate $\int \frac{1}{\sqrt{x+1} + \sqrt{x-1}} dx$.

Solution: Given $\int \frac{1}{\sqrt{1+x} + \sqrt{x-1}}$

On rationalising we get,

$\int \frac{1}{\sqrt{1+x} + \sqrt{x-1}} \times \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x+1}}$

$$= \int \frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x+1}} \times \frac{1}{\sqrt{x+1}-\sqrt{x-1}}$$

$$= \frac{\sqrt{x+1}-\sqrt{x-1}}{(x+1)-(x-1)}$$

$$= \frac{\sqrt{x+1}-\sqrt{x-1}}{\cancel{x}+1-\cancel{x}+1} = \frac{\sqrt{1+x}-\sqrt{x-1}}{2}$$

$$= \frac{\sqrt{x+1}-\sqrt{x-1}}{2}$$

$$= \frac{1}{2}\,(x+1)^{\frac{1}{2}}-(x-1)^{\frac{1}{2}}$$

$$= \frac{1}{2}\,\frac{(x+1)^{\frac{1}{2}-1}}{\left(\frac{1}{2}\right)} - \frac{(x-1)^{-\frac{1}{2}}}{-\frac{1}{2}}$$

$$= \cancel{\frac{1}{2}}\cancel{2}\frac{1}{\sqrt{x+1}} + \cancel{\frac{1}{2}}\frac{1}{\sqrt{x-1}}$$

$$= \frac{1}{\sqrt{x-1}}$$

On integrating we get,

$$I = \frac{1}{2}\left[(x+1)^{\frac{1}{2}} - (x-1)^{\frac{1}{2}}\right]$$

$$= \frac{1}{2}\left[\frac{(x+1)^{3/2}}{\frac{3}{2}} - \frac{(x-1)^{3/2}}{3/2}\right]$$

$$= \frac{1}{3}\left[(x+1)^{3/2} - (x-1)^{3/2}\right]$$

Ans